INTERLAMINAR PLASTICITY IN COMPOSITE LAMINATES: MODELLING AND COMPUTATION

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Abstract
In this paper, a model of laminated plates called M4-5N and validated in a previous paper is modified in order to take into account plasticity at the interfaces between layers. Displacement discontinuities at the interfaces are considered in the model. These discontinuities are calculated by means of a plasticity model which has non linear equations. In order to compute the model, the LATIN method is employed. The method is then applied to the resolution of a free edge problem of a composite laminate. The calculations and a pertinent delamination criterion are used to predict delamination onset in a family of carbon-epoxy laminates. A good agreement between the theoretical and experimental results is shown.

1. Introduction

Delamination is perhaps the most critical failure mode in laminated structures. It is generally due to the stress concentration near the edges at the interfaces between layers. This concentration provokes usually non linear phenomena such as plasticity before delamination onset. Thus, it is necessary to take into account interlaminar plasticity in the stresses calculations in order to predict accurately delamination onset. For these calculations, 3D finite elements may be used but this technique has a high computational cost. The presence of singularities complicates the stress analysis and the determination of a pertinent delamination criterion. In this paper, a layerwise model called M4-5N [1] is used to calculate the interlaminar stresses. This model is similar to Pagano's local model [2] and exhibits finite results even at the intersection of the interfaces with the edges. The model has already been validated for thermal elastic problems [3] and has been implemented in the software called DEILAM developed by Díaz et al [4]. Herein, in order to take account of plasticity at the interfaces, a plasticity model is adapted and introduced into the M4-5N model. The laminate is supposed to be made up of linear elastic layers and elastic-plastic interfaces. The LATIN method [5] is then used for the numerical resolution. Finally, the calculations and a maximum sliding criterion are applied to predict delamination onset for the family of carbon-epoxy laminates tested by Díaz and Caron [6].

2. Model equations

By using the M4-5N model, the multi-layer (3-D object, as shown in figure 1) becomes a superposition of N Reissner plates [7] (a 2-D object with N particles at each geometrical point and for each particle 5 kinematic fields are considered) coupled with interlaminar stresses. This model has been inspired by Pagano's work [3] and already developed and validated in its linear elastic version [1,3]. The model takes into account displacement discontinuities at the interfaces between layers [4]; these fields are supposed to be known. Since these discontinuity fields are given data, the constitutive equations of the model are linear elastic. Herein, the interface is modeled as a thin layer within which plasticity may occur and the displacement discontinuities depend on the amount of plasticity in the interface. The plasticity model chosen in this paper uses a Von-Mises yield function, a hardening (or softening) function and a normal flow rule.

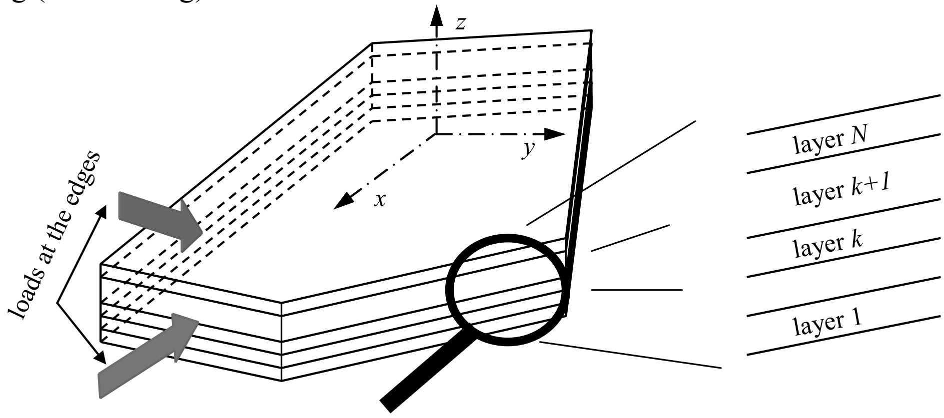

Figure 1. Considered laminate

Since the linear elastic equations of the M4-5N model have already been developed in [1,3,4], in this section we only show the constitutive equations in which the interlaminar discontinuities appear. These are the generalized elastic constitutive equations related to:

- the shear stresses at interface *j,j*+1 between layers *j* and *j*+1 ($1 \le j \le N-1$)

$$\begin{aligned} D_\alpha^{j,j+1} - \gamma_\alpha^{j,j+1} &= -\frac{1}{10} S_{Q\alpha\beta}^{j} Q_\beta^{j} - \frac{1}{10} S_{Q\alpha\beta}^{j+1} Q_\beta^{j+1} - \frac{t^j}{30} S_{Q\alpha\beta}^{j} \tau_\beta^{j-1,j} \\ &+ \frac{2}{15}\left(t^j S_{Q\alpha\beta}^{j} + t^{j+1} S_{Q\alpha\beta}^{j+1}\right)\tau_\beta^{j,j+1} - \frac{t^{j+1}}{30} S_{Q\alpha\beta}^{j+1} \tau_\beta^{j,j+1} \end{aligned} \tag{1}$$

- the normal stresses at interface *j,j*+1 between layers *j* and *j*+1 ($1 \le j \le N-1$)

$$\begin{aligned} D_3^{j,j+1} - \gamma_3^{j,j+1} &= \frac{9}{70} t^j S_3^j \nu^{j-1,j} + \frac{13}{35}\left(t^j S_3^j + t^{j+1} S_3^{j+1}\right)\nu^{j,j+1} \\ &+ \frac{9}{70} t^{j+1} S_3^{j+1} \nu^{j+1,j+2} \end{aligned} \tag{2}$$

where $(\alpha,\beta) \in \{1,2\}^2$, $D_\alpha^{j,j+1}$ and $D_3^{j,j+1}$ are the generalized interlaminar strains, Q_β^j is the generalized shear force in layer *j* in direction *β*, t^j is the thickness of layer *j*, $\tau_\beta^{j,j+1}$ and $\nu^{j,j+1}$ are the generalized interlaminar shear and normal stresses, respectively, $\gamma_\alpha^{j,j+1}$ and $\gamma_3^{j,j+1}$ are the interlaminar in-plane and out-of-plane displacement discontinuities, respectively. The displacement discontinuities are supposed to be known during the development of the M4-5N equations. Let us now write the equations that will help to determine these displacement discontinuities.

The interface is considered as a thin layer which has an elastic-plastic behavior. We assume that all the stresses in this layer are negligible except the out-of plane stresses ($\tau_\beta^{j,j+1}$ and $\nu^{j,j+1}$). The elastic behavior of interface *j,j*+1 between layers *j* and *j*+1 subjected to a shear stress $\tau_1^{j,j+1}$ is written as follows :

$$\gamma_1^{j,j+1}(x,y) - \gamma_1^{j,j+1,p}(x,y) = 2e^{\text{int}} \frac{\tau_1^{j,j+1}(x,y)}{G^{\text{int}}} \tag{3}$$

where $\gamma_1^{j,j+1,p}$ is a plastic sliding, G^{int} is the shear modulus of the material. The two other elastic equations (in directions *y* and *z*) are similar to the previous one. The plasticity yield function is :

$$\begin{gathered} \sqrt{3\left(\tau_1^{j,j+1}\right)^2 + 3\left(\tau_2^{j,j+1}\right)^2 + \left(\nu^{j,j+1}\right)^2} - \sigma^0 - K^{j,j+1}\left(p^{j,j+1}\right)^{\alpha^{j,j+1}} = 0 \\ p^{j,j+1} = \frac{1}{e^{\text{int}}}\sqrt{\frac{1}{3}\left(2\left(\dot{\gamma}_1^{j,j+1}\right)^2 + 2\left(\dot{\gamma}_2^{j,j+1}\right)^2 + \left(\dot{\gamma}_3^{j,j+1}\right)^2\right)} \end{gathered} \tag{4}$$

where $p^{j,j+1}$ is the cumulative plastic strain at interface *j,j*+1, $K^{j,j+1}$ and $\alpha^{j,j+1}$ are properties of the interface. The flow rules are similar to that of $\gamma_1^{j,j+1,p}$:

$$\dot{\gamma}_1^{j,j+1,p} = 3e^{\text{int}}\dot{p}^{j,j+1} \frac{\tau_1^{j,j+1}}{\left(\sigma^0 + K^{j,j+1}\left(p^{j,j+1}\right)^{\alpha^{j,j+1}}\right)} \tag{5}$$

3. Numerical resolution

The laminate is subjected at its boundaries to a mechanical load. Before reaching its value, the mechanical load takes intermediate values called load steps. In order to solve the equations for each load step, the LATIN (LArge Time Increment) method employed by Allix and Vidal [5] is applied. The linear equations and the non linear equations are separated. Two sub-problems are then considered: the first is non linear (problem A) and the other is linear (problem B).

After the *n*-th iteration, in problem A, a provisional solution $s_{B,n} = \left(\tilde{F}_{B,n}, \tilde{\gamma}_{B,n}^{j,j+1}, p_{B,n}^{j,j+1}\right)$ of problem B is transformed into a new provisional and more accurate solution $s_{A,n+1} = \left(\tilde{F}_{A,n+1}, \tilde{\gamma}_{A,n+1}^{j,j+1}, p_{A,n+1}^{j,j+1}\right)$ which verifies equations (3-5) and $\tilde{F}_{A,n+1} = \tilde{F}_{B,n}$; where $\tilde{F}$ and $\tilde{\gamma}$ are the vectors of interlaminar stresses and displacement discontinuities. In problem B, the provisional solution $s_{A,n+1} = \left(\tilde{F}_{A,n+1}, \tilde{\gamma}_{A,n+1}^{j,j+1}, p_{A,n+1}^{j,j+1}\right)$ of problem A is considered and a new provisional and more accurate solution $s_{B,n+1} = \left(\tilde{F}_{B,n+1}, \tilde{\gamma}_{B,n+1}^{j,j+1}, p_{B,n+1}^{j,j+1}\right)$ is calculated. In this linear problem, equations (3-5) are linearized by means of a simple differentiation and the software DEILAM [4] is employed for solving the M4-5N equations by making use of a finite element resolution. After a few iterations, calculations converge for the considered load step.

4. Application example

Let us apply our theoretical tool to the experimental observations and results found by Díaz and Caron in [6]. The previous authors carried out tensile tests with $\left(\pm 10_n\right)_s$ and $\left(\pm 20_n\right)_s$ carbon-epoxy laminates. During the tests they observed an interlaminar sliding at the interfaces between layers θ and $-\theta$ as shown in figure 2. For all the laminates, delamination appeared at almost the same value of the interlaminar sliding: $\gamma = 15\mu\text{m}$. Herein, we use this critical interlaminar sliding as a delamination criterion. The properties of the interface that appear in our model were determined as follows:

- the thickness of the interface is 3µm (measured by Díaz and Caron [6])

- the elastic properties of the interface are E=5GPa and ν=0.3 (these are typical values of an epoxy matrix)
- the plastic properties were determined by the method of least squares in order to provide the best possible predictions for all the tested laminates.

In figure 3, the critical loads predicted by our model are compared to the experimental results. We can see that our predictions are very accurate.

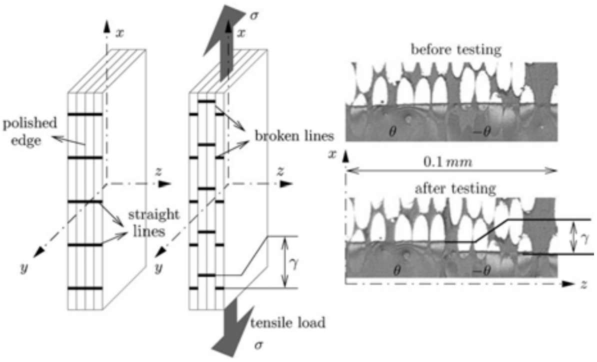

Figure 2. Revealing interlaminar sliding

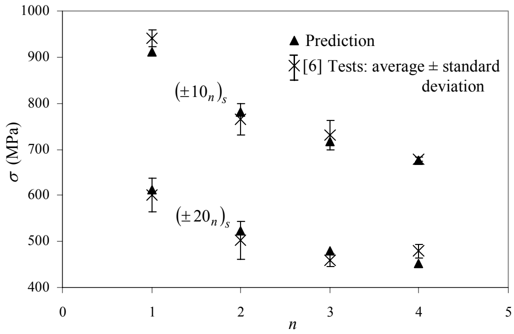

Figure 3. Critical loads. Predictions and tests

5. Conclusions

In conclusion, a theoretical tool has been developed to calculate the interlaminar plasticity in laminated structures. A model of interlaminar plasticity has been introduced into the model of laminated plates called M4-5N. The nonlinear equations of the model are solved by means of a numerical technique based on the LATIN method. One application example showed that the theoretical tool can help predict delamination onset.

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