Influence of the spatial non-uniformity of the grating on the beam energy exchange of thick sillenite crystals.

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Abstract: The influence of the variation of the phase and magnitude of the complex light modulation along sample thickness on the gain in a thick crystal of BTO in two-wave mixing was obtained.

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1. Introduction

When energy exchange takes place between two waves coupled by a photorefractive phase grating in a strong non-linear regime the light modulation does not remain constant along wave propagation and there is a spatial non-uniform grating along sample thickness. The undepleted approximation (uniform grating) is valid for short interaction lengths and or small coupling. However, for thick samples, the interaction length of the recording light beams inside the photorefractive material already is not small and therefore the approximation of uniform grating is not valid.

Here we calculate the influence on the beams energy exchange of the variation, along sample thickness, of the magnitude and phase of the light modulation in a transmission thick sillenite grating (BTO) recorded with a dc applied field of E_0 = 20 kV/cm, an initial light modulation of $m_0 = 0.9$ when the grating vector is parallel to the face (001) and the light waves are propagating in the $(\bar{1}10)$ plane. Birefringence k_0 and absorption of light (α=0.6 cm^{-1}) are considered.

2. Theoretical model

We solved numerically the set of non-linear material rate differential equations to obtain the variation of the overall space charge field as a function of several initial light modulations considering an applied electric field [1]. Then we followed a tensor approach to express in terms of the complex amplitudes the four coupling equations (1a)-(1d) that describe the propagation and energy exchange of the components of the light waves $\vec{R}(z) = R(z)\exp(-i\vec{k}_R \cdot \vec{r} + \Psi_R)$ and $\vec{S}(z) = S(z)\exp(-i\vec{k}_S \cdot \vec{r} + \Psi_S)$ along the sample thickness [2]. Describing the wave vectors $\vec{k}_R$ and $\vec{k}_S$ the direction of propagation of $\vec{R}(z)$ and $\vec{S}(z)$ respectively and Ψ_R , and Ψ_S representing the phases of the two light waves. In fact, $\vec{R}(z)$ and $\vec{S}(z)$ can be written as a function of their complex amplitudes R_E, S_E ,R_M, and S_M along the direction $\hat{u}_E$ perpendicular to the x-y plane and $\hat{u}_M$ parallel to the same plane as follows: $\vec{R}(z) = R_E(z)\hat{u}_E + R_M(z)\hat{u}_M$ $\quad \vec{S}(z) = S_E(z)\hat{u}_E + S_M(z)\hat{u}_M$; The incidence of the beams is on the *x-y* plane.

Neglecting the second derivate of the field and considering the paraxial approximation, the following set of equations for the grating wave vector $K_G \,||\,$ [001] with the light waves propagating in the $(\bar{1}10)$ plane, is obtained [3]:

$$\frac{dR_M(z)}{dz} = -\rho R_E(z) - \frac{\alpha}{2} R_M(z) \tag{1a}$$

$$\frac{dR_E(z)}{dz} = \rho R_M(z) + i\kappa_0 R_E(z) + i\kappa_1^*(z) S_E(z) - \frac{\alpha}{2} R_E(z) \tag{1b}$$

$$\frac{dS_M(z)}{dz} = -\rho S_E(z) - \frac{\alpha}{2} S_M(z) \tag{1c}$$

$$\frac{dS_E(z)}{dz} = \rho S_M(z) + i\kappa_0 S_E(z) + i\kappa_1(z) R_E(z) - \frac{\alpha}{2} S_E(z) \tag{1d}$$

Here α is the absorption coefficient and ρ is the optical activity which is neglected in this work for simplicity. The constant κ_0, is due to the variation of the magnitude of the change in the refractive index induced by the external applied field, E_0:

$$\kappa_0 = \frac{2\pi\Delta n_0}{\lambda\cos\theta} \tag{2}$$

where

$$\Delta n_0 = \frac{n_0^3 r E_0}{2} \tag{3}$$

n_0 is the average refraction index in the sample, λ is the wave length of the recording monochromatic beams, θ is the incidence Bragg's angle and r is the electro optic coefficient. Notice that κ_0 is not a function of z.

In our approach we take into account in the complex coupling factor $\kappa_1(z)$ of the two-wave mixing equations the phase shift $\phi(z)$ of the space charge field with regard to the light interference pattern as well as the phase $\psi_m(z)$ of the light modulation $m(z)$ determined from:

$$e^{i\psi_m(z)} = \frac{m(z)}{|m(z)|}\text{ , so that:}$$

$$\kappa_1(z) = \frac{\pi}{\lambda\cos\theta}\frac{n_0^3 r|E_1(z)|}{2}\exp i(\phi(z)+\psi_m(z)) = \frac{\pi\Delta n_1(z)}{\lambda\cos\theta} \tag{4}$$

where E_1(z) is the fundamental Fourier component of the space charge field.

In terms of the complex amplitudes of light beams, the light modulation is described by

$$m(z) = 2\frac{S_E(z)R_E(z)^* + S_M(z)^* R_M(z)}{(R_E(0)R_E^*(0) + R_M(0)R_M^*(0) + S_E(0)S_E^*(0) + S_M(0)S_M^*(0))} \tag{5}$$

We solved the set of equations (1a)-(1d) in a self consistent way to take into account the variation with depth of the refraction index given by Δn_1(z). We divided the sample in thin layers of thickness Δz in such a way that within each layer $\kappa_1(z)$ is practically constant. In this way, within each layer we have analytical solutions for the coupled equations (1a)-(1d) [4]. When a small change (larger than 0.1%) in this variable occurred, we chose a smaller interval and calculated the new corresponding set of values of constants for the corresponding interval Δz. We started evaluating the initial set of constants for the first layer at the surface of the sample by using $\kappa_1(z=0)$. Next, for the following layers, the values of the complex amplitudes of the beams at the end of each interval were used to evaluate m (z) and therefore a new value of κ_1 at z where the following layer starts.

In our calculations we used a grating with a spatial period Λ of 10μ, a light modulation at the surface of the sample (m_0) of 0.9 and an applied field of 20 kV/cm, obtaining a coupling factor of $\kappa_1(0) = 4.79\ cm^{-1}$. The value of absorption used for the BTO crystal was 0.6 cm^{-1}, the rest of material parameters were taken as in Refs. [3,5]. We also considered that the two beams were linearly polarized and had a polarization angle ϕ_p, defined as the inclination angle of the electric field of light waves with respect to the plane of incidence at the surface of the sample, of $\pi/2$ rad.

From the complex amplitudes of light waves, obtained from the self consistent solutions of the set of equations (1a)-(1d) we calculate the intensities and phases of each wave and the corresponding light modulation m(z) as a function of z. We also obtained the gain coefficient G(z) defined as:

$$G(z) = \frac{I_R(z)}{I_R(z=0)} - 1 \tag{6}$$

Where $I_R(z) = |R(z)|^2$ is the intensity of the reference light beam at specific sample thickness z, and I_R(z=0) is the intensity of light beam at the surface of the sample.

3. Results and discussion

In figure 1 we compare the variation of the phase of the light modulation along sample thickness of the uniform grating approximation (□ α =0 and **x** α =0.6cm^{-1}) with the non-uniform grating approximation (◆ α =0 and Δ α =0.6cm^{-1}). In figure 2 we show the corresponding gain associated to figure 1 for the uniform grating (■ α =0 and **x** α =0.6cm^{-1} *)* and the non-uniform grating approximation *(*◆ α =0 and Δ α =0.6cm^{-1}).
For the uniform grating approximation the phase of the light modulation has an identical oscillatory behavior for both cases considered α =0 and α =0.6cm^{-1}. Instead for the non-uniform grating calculations, in the case of no absorption α = 0 the phase of the light modulation increases monotonously along the sample thickness, while in the case of α =0.6cm^{-1} also it increases but tends to reach a saturation value. On the other hand, the gain coefficient of the recording beams in the case of uniform grating also shows an oscillatory behavior as the phase of the modulation, becoming attenuated by absorption. In contrast, the gain coefficient for the non uniform grating as can be seen in figure 2 increases until reaching a saturation value in the case of no absorption but calculations for α =0.6cm^{-1} show a maximum value of the gain coefficient at specific thickness z followed by a monotonously decrement along the sample thickness.

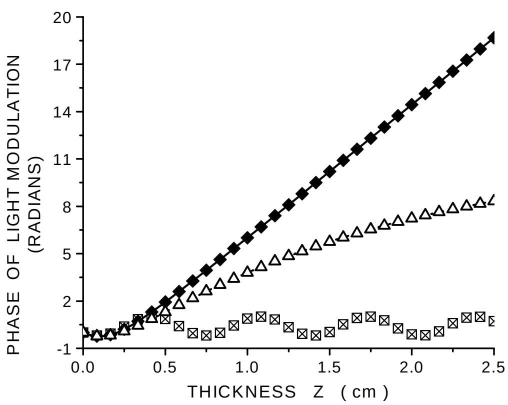

Fig. 1 Variation of the phase of the light modulation along sample thickness for the uniform grating approximation without absorption □ α = 0 and **x** α= 0.6 cm^{-1}, and for the non uniform grating approximation ♦ α= 0 and Δ α= 0.6 cm^{-1}.

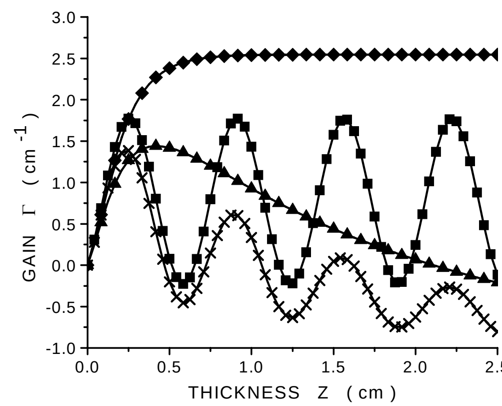

Fig. 2 Gain coefficient G(z) along the sample thickness for the uniform grating approximation ■ α = 0 and **x** α= 0.6 cm^{-1}, and for the non uniform grating approximation ♦ α= 0 and Δ α= 0.6 cm^{-1}.

4. Conclusions and Acknowledgment

In conclusion, the vector approach employed in this work to describe the propagation and energy exchange of the components of the recording light waves in the material predicts strong variations of the phase and the magnitude of the modulation along the thickness sample in the non uniform grating approximation. This variation of the complex light modulation modifies significantly the gain in the photorefractive recording process.
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