# CAVITY BUBBLE RADIUS OF CURVATURE

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*SUMMARY*

Onset of nucleate boiling on a heat transfer surface at moderated temperatures (heterogeneous nucleation) requires the existence of bubble embryos trapped inside surface cavities. A concave vapour bubble in thermo-mechanical equilibrium requires the surrounding liquid to be in a superheated state. A convex vapour bubble requires the surrounding liquid to be subcooled. At the liquid-solid-vapour interface, the surface orientation and the liquid-solid contact angle dictate the orientation of the bubble radius of curvature. If some bubble embryos are to survive subcooled conditions during non-boiling periods, their cavity inner geometry have to be re-entrant. Poor wetting substances have better chances to survive non-boiling than well wetting ones.

## I. INTRODUCTION

The formation of bubbles at the *onset of nucleate boiling* (ONB) and during fully developed nucleate boiling has been known to occur from small cavities in the heat transfer solid surface since the 1950's (Bankoff [1], Corty and Foust [2], Clark *et al.* [3]). For poor wetting substances, like water, the transition from convection to nucleate boiling is usually smooth as the wall heat flux increases. With well wetting substances, like refrigerants, an abnormal departure from the *normal* boiling curve, a *temperature overshoot* (TOS) builds up before the ONB, driving the wall temperature to higher values than expected, only to drop to its *normal* values when the nucleate boiling mechanism erupts on the heat transfer surface (Hino and Ueda [4], Kim and Bergles [5], McDonald and Shivprasad [6], You *et al.* [7]).

The main purpose of this paper is to show, through a simple geometric analysis, that the different behaviour of the poor wetting and well wetting substances on the same heat transfer surface, at the ONB, is caused by the substance contact angle. The paper focuses on the effects of the interaction between the solid surface and the bubble embryo on the bubble radius of curvature. The objective is to determine what kind of cavity geometries could maintain a bubble embryo in thermo-mechanical equilibrium when the external pressure or temperature fields vary.

## II. BUBBLE THERMO-MECHANICAL EQUILIBRIUM

For a bubble in thermal equilibrium with the surrounding liquid, the relationship between the pressure inside ($P_g$) to the pressure outside ($P_f$), required to maintain also a mechanical equilibrium, is given by:

$$P_g = P_f + \frac{2\,\sigma}{r} \tag{1}$$

Where $\sigma$ is the surface tension and r is the bubble radius of curvature. At this equilibrium condition the surrounding liquid is in a superheated thermodynamic state, and the degree of superheat *required* in the liquid to maintain the bubble in equilibrium (RLS) is given as:

$$\begin{aligned} RLS &= T_s(P_g) - T_s(P_f) \\ &= T_s\left(P_f + \frac{2\,\sigma}{r}\right) - T_s(P_f) \end{aligned} \tag{2}$$

Thermo-mechanical equilibrium of a bubble can exist only when the *actual liquid superheat* (ALS) and the bubble's RLS are equal. This means that for a given liquid pressure and temperature, only bubbles of one particular radius of curvature are in thermo-mechanical equilibrium. The reaction of a bubble to any temperature or pressure perturbation would be a change of its volume and consequently, a change of its radius of curvature and RLS. An increase in the ALS, as a consequence of an increase on the liquid temperature (or a decrease on the liquid pressure), induces liquid vaporization at the bubble surface, and therefore an increase in the bubble's volume. A decrease of the ALS, as a consequence of a decrease on the liquid temperature (or an increase of the liquid pressure), induces vapour condensation in the bubble. If a bubble is in the liquid bulk, its radius of curvature *increases* as the bubble volume *increases*, and vice versa. Such a bubble cannot maintain its thermo-mechanical equilibrium and will either grow or shrink continuously after any pressure or temperature perturbation. On the other hand, the interaction of a bubble with a solid surface may cause the bubble radius of curvature to *shrink* as the bubble volume *increases*, thus permitting the bubble to survive.

### *A. Bubble Embryo Dormancy*

Bubble nucleation does not occur in the bulk liquid (homogeneous nucleation), but rather in surface cavities and scratches (heterogeneous nucleation). This has been shown theoretically by Bankoff [1], and experimentally by Clark *et al.* [3] among others.

Since temperatures needed to reach the RLS required for homogeneous nucleation (Lienhard, [8]) are much higher than experimental results reported in the literature (Murphy and Bergles [9], Yin [10], Hino and Ueda [4], Kim and Bergles [5]), it follows that heterogeneous nucleation is the controlling process where bubbles must initiate from pre-existing bubble embryos within cavities.

The survival of small bubbles (bubble embryos) trapped inside cavities during non-boiling periods, with saturation or even subcooled conditions, is required to reactivate the nucleate boiling mechanism at the heat transfer surface. For substances with a large contact angle, like water, the onset (reactivation) of nucleate boiling (ONB) on a heat transfer surface has been observed to occur readily at RLS within 2 or 3 K of values predicted by the normal boiling curve (Bergles and Chyu [11], Sudo *et al.* [12]). With well wetting substances (small contact angle), like refrigerants, liquid metals and some organic substances, it has been reported by numerous researchers the occurrence of temperature overshoots (TOS) at the ONB, as large as 20 to 30 K (Hino and Ueda [4], Kim and Bergles [5], McDonald and Shivprasad [6]) and up to 53 K by You *et al.* [7]. From these results it can be inferred that only very small bubble embryos survived within cavities, and that the contact angle and cavity geometry must play an important role in the size of the surviving embryos.

This paper investigates the behaviour of the bubble radius of curvature in cavities in order to determine the cavity geometric characteristics that would permit a bubble to move to a new equilibrium condition when the liquid pressure or temperature are perturbed. This persisting equilibrium can be accomplished only if the bubble radius of curvature decreases as the bubble volume increases, and vice versa.

## III. CAVITY GEOMETRY

In order to numerically investigate the "mechanical" behaviour of a bubble in a cavity it is necessary to consider a specific class of cavity geometries. The simple but general shape shown in Fig. 1 was selected for investigation. The outer portion, if present, is an axisymmetric diverging cone. The inner portion, if present, is, at its smallest point, an axisymmetric converging cone which can have any subsequent shape to close the cavity. This

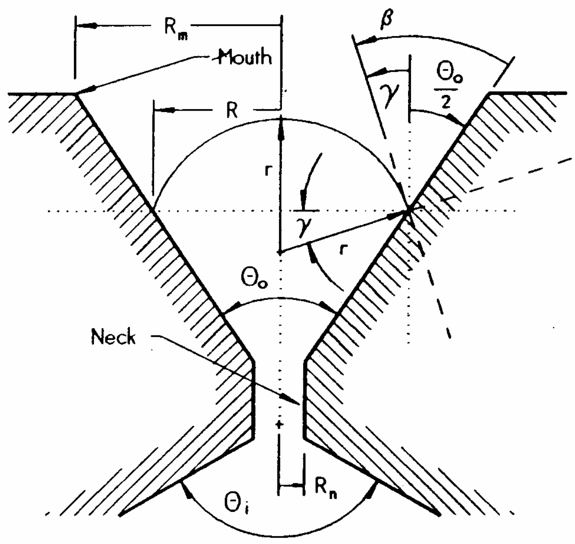


Figure 1. Geometry of a bubble in a conical cavity.

simple generalized cavity shape allowed the phenomenon to be mathematically modelled and allows the conclusions to be readily applied to other shapes.

### *A. Bubble Geometry in a Divergent Conical Cavity*

A small bubble attached to a solid surface has a liquid-vapour interface which is essentially a spherical segment. For a given liquid-solid-vapour combination, the angle formed by the liquid with respect to the solid surface, at the bubble triple interface, is referred to as the *Contact Angle* (ß). The contact angle of a stationary bubble, for a given liquid-solid combination, is a function only of temperature (which is presumed constant in the vicinity of the bubble) and hence the angle between the tangent to the bubble at the point of surface contact and the axis of symmetry of the cavity is a function only of the solid surface orientation at the triple interface point.

A bubble attached to the walls of a divergent conical cavity is depicted in Fig. 2. Based on this figure, an analytical expression can be derived to calculate the bubble radius of curvature (r) as a function of the cavity angle ($\Theta_o$), contact angle (ß) and the horizontal radius between the cavity axis and the contact point between the bubble and the cavity wall (R).

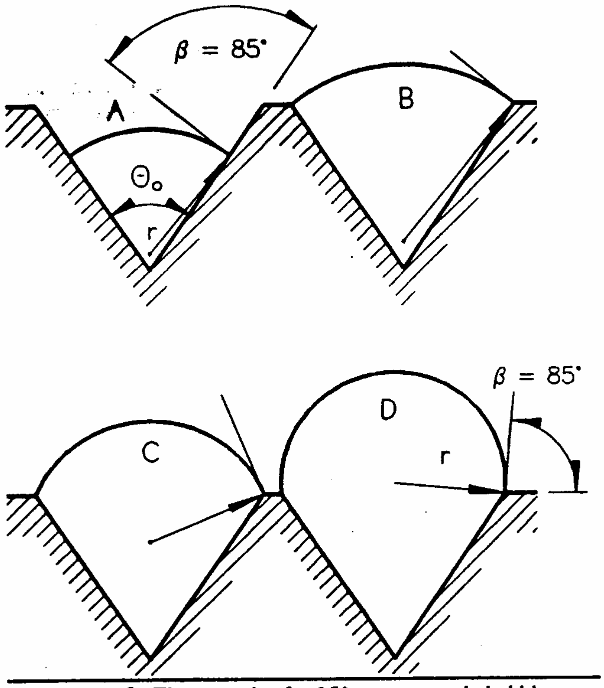


Figure 2. The growth of a 85° contact angle bubble.

A bubble attached to the inner walls of a conical cavity "sees" a constant angle $\gamma$ until it reaches the mouth. During the bubble growth process within the cavity (increasing trapped vapour volume), the bubble radius of curvature increases linearly with R. As the bubble radius of curvature increases, the pressure within the bubble, and the corresponding saturation temperature, decrease, (see eq. 1) thus reducing the bubble's RLS. For a constant wall temperature, this means that the ALS of the adjacent liquid becomes larger than the bubble's RLS. This condition causes liquid to vaporize into the bubble promoting further growth. The net result is a self sustained bubble growth until either the bubble attaches at the mouth or is expelled from the cavity. Equilibrium is possible only at the cavity mouth.

Since we want to analyze the behaviour of a bubble inside a cavity, at the cavity mouth and outside of the cavity, and since the shape and size of the bubble depend basically on the orientation of the solid surface at the triple interface line, in the remainder of this paper the term *cavity angle* will have a broader meaning. The cavity mouth is the boundary between the cavity and the external surface. Therefore from the point of view of a bubble growing at the cavity mouth, the orientation of the solid surface changes from the cavity angle up to 180°. Cavity angle means the angle of the walls of an imaginary cavity that is tangent to the solid surface at the triple interface line. With this definition, at the cavity mouth we will consider that the bubble *sees* how the cavity angle goes from whatever value it had inside the cavity up to the 180° of the external surface.

A growing bubble attached to the cavity mouth will continue to increase *in volume* as its contact angle with respect to the external surface becomes equal to its normal contact angle. During this process the *radius of curvature* will change by rotating over the cavity mouth as the bubble expands, as can be observed with bubbles B, C and D in Fig. 2. Once this is achieved the bubble will spread outside of the cavity mouth.

When the contact angle is greater than half the cavity angle, a bubble *growing* out of the mouth of a cavity will undergo a *reduction* of its radius of curvature. This case is shown in Fig. 2 for a bubble contact angle of 85°. It can be observed that the radius of curvature, for this particular case, has to first shrink and then grow during bubble growth from curve B to D, and that means a possible equilibrium condition at the cavity mouth.

When the contact angle is less than half the cavity angle, (as shown in Fig. 3), the radius of curvature will *increase* as the bubble *expands* out of the mouth of the cavity. Such a fluid/cavity combination will not find thermo-mechanical equilibrium along the cavity walls or at the cavity mouth.

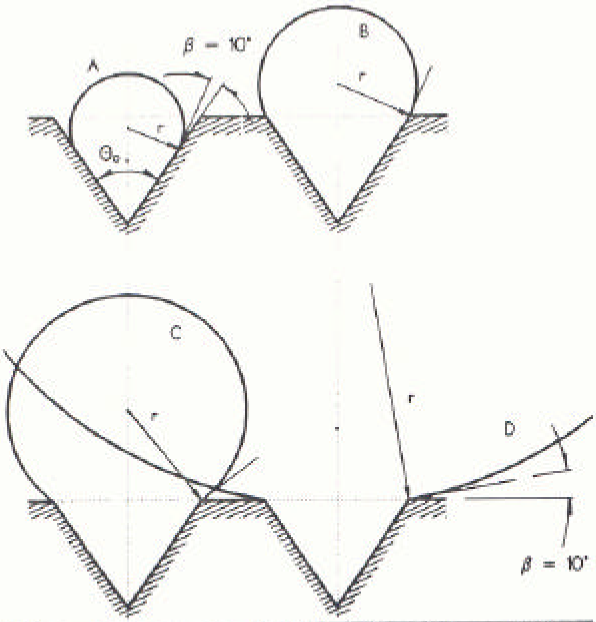


Figure 3. The growth of a bubble at the cavity mouth for a 10° contact angle.

## IV. BUBBLE RADIUS OF CURVATURE

The relationship between the bubble radius of curvature and the contact angle, the cavity angle and the distance R, is given by:

$$r = \frac{R}{\cos(\gamma)} \tag{3}$$

Where:

$$\gamma = \beta - \frac{\theta}{2} \tag{4}$$

ß is the solid/liquid contact angle, and Θ varies from $\Theta_o$ to $\pi$ at $R/R_m = 1$:

$$\theta = \begin{cases} \theta_o & \forall \quad 0 < \dfrac{R}{R_m} < 1 \\ \pi & \forall \quad \dfrac{R}{R_m} > 1 \end{cases} \tag{5}$$

The angle Gamma (γ) is a constant for a bubble *inside* a conical cavity. Gamma increases as the bubble grows while attached to the *cavity mouth*, and then remains constant as the bubble spreads *outside* of the cavity mouth.

The bubble radius of curvature increases linearly as a bubble grows within a divergent conical cavity. For a cavity with an angle equal to twice the contact angle, $\Theta_m = 2ß$, the bubbles would have a minimum radius of curvature, r = R. Cavities where $\Theta < \Theta_m$ will contain bubbles which have the centre of their radius of curvature displaced along the axis of rotation of the cavity toward the apex from the minimum radius point (eg Fig. 3). Cavities of more than twice the contact angle will contain bubbles which have the centre of their radius of curvature displaced along the axis of rotation of the cavity further away from the apex of the cavity (eg Fig. 2). At the mouth of a cavity where $\Theta < \Theta_m$, the radius of curvature of a bubble must first decrease to equal $R_m$ before it grows out of the cavity. No such decrease occurs for $\Theta > \Theta_m$ degree cavities.

### *A. The Evolution of the Radius of Curvature During the Growth Process*

The bubble radius of curvature size evolution of a bubble that starts growing deep inside of a conical cavity, of a given cavity angle, reaches the cavity mouth and finally outgrows it and spreads over the outside flat surface will be now compared for substances with contact angles of 10° and 85°, which could be the cases of a commercial refrigerant and water respectively. Fig. 4 shows a comparison where the cavity angle is 20°. It is possible to observe the following relevant points:

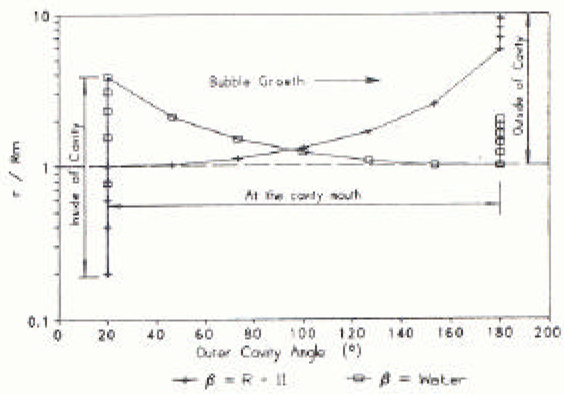


Figure 4. Bubble curvature radius evolution over a 20° cavity.

- When the bubbles first reach the mouth of a typical cavity, the radius of curvature of the 10° substance is approximately the same size as the cavity mouth radius, whereas for the 85° substance radius of curvature is substantially larger.
- During the growing process at the cavity mouth the radius of curvature of the small contact angle substance *increases* to six times the cavity mouth radius, while the radius of curvature of the large contact angle substance *reduces* to the same size as the cavity mouth radius.
- Both substances experience an increase in their radius of curvature as the bubble grows beyond the cavity mouth.

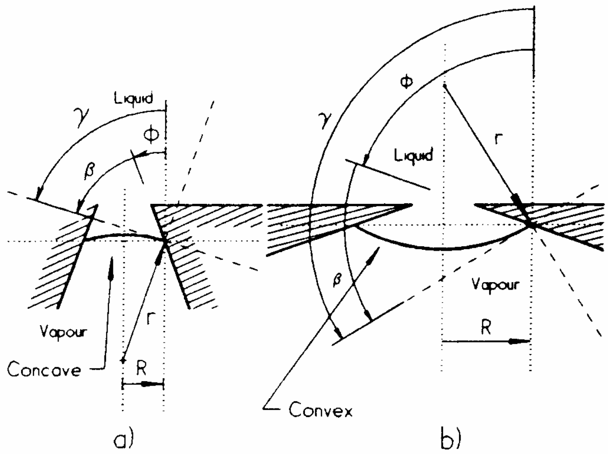


Figure 5. Re-entrant cavity shapes.

## V. THE RE-ENTRANT CAVITY

From previous sections we may conclude that in a divergent conical cavity the bubble radius of curvature reduces as a bubble contracts into the cavity. Bubbles of smaller radius of curvature require larger liquid superheats (higher temperatures) to maintain their equilibrium and, therefore, a bubble that shrinks into a divergent conical cavity will collapse and disappear unless its radius of curvature stops shrinking. This can only be brought about by a change in the orientation of the cavity walls such as a change from a divergent conical shape to a convergent conical cavity as shown in Fig. 5. Cavities with such combined shapes are usually referred to as re-entrant cavities.

The cavity angle is defined to be positive for divergent conical cavities and negative for convergent conical cavities. In order to avoid a discontinuity in the description of the surface angle at a sharp edge, the surface at that point will be assumed to have an infinitesimal radius.

Fig. 5 shows examples of re-entrant cavities. Case (a) shows a bubble with a positive radius of curvature and case (b) shows a cavity of larger negative $\phi$, for which the bubble radius of curvature becomes *negative*. A negative radius of curvature means that the bubble is now convex with respect to the vapour phase and the surface tension forces act against the liquid phase. In both these cases the radius of curvature increases as the bubble recedes into the convergent conical portion of the cavity.

From Fig. 5 it is readily obvious that for some combinations of cavity angle and liquid contact angle $\beta$ the vapour-liquid interface becomes flat. This occurs when $\gamma = \beta - \phi = 90°$. When this condition occurs, the pressures on both sides of the bubble interface are the same and the thermodynamic states are those of saturation for both phases. If the bubble radius of curvature becomes negative the surface tension forces compress the liquid phase. Such a cavity will permit the vapour to remain saturated and in equilibrium with *subcooled* liquid.

It should be noted that the minimum possible positive value for the bubble radius of curvature occurs at the edge of the re-entrant neck. For a given fluid, if the cavity angle is sufficiently negative to create a convex bubble (negative bubble radius of curvature), then the bubble radius of curvature will also attain a maximum positive value at the re-entrant neck as the bubble surface becomes flat when the radius of curvature pivots at the re-entrant neck, and then will turn negative as the bubble surface changes to convex. After this point, the bubble radius of curvature will keep reducing its negative magnitude until the contact angle acquires its normal value with respect to the inner cavity walls. As the bubble size continues to shrink the radius of curvature increases in magnitude. Note that:

- The *minimum* bubble radius of curvature occurs when the bubble is at the re-entrant neck. The minimum concave radius of curvature is equal to the neck radius. The minimum convex radius of curvature occurs when the bubble is about to recede into the cavity below the neck and will always be greater than the neck radius.
- The *minimum concave* value of the radius of curvature will determine the superheat required for a bubble to depart from the neck, (ie. the

minimum required superheat to activate such a cavity).

- The *minimum convex* radius of curvature will dictate the maximum subcooling that the bubble embryo could resist before the liquid floods (quenches) the cavity.

For a given re-entrant cavity neck radius, the angles of the surfaces above and below the neck will determine the shape and radius of curvature of a bubble at the point where the bubble grows outside of the cavity neck. These conditions are shown in Fig. 6, where in the top row of figures bubbles of 10° and 85° of contact angle are shown with the shapes that they acquire just before sliding out and into the cavity.

At this point it is very useful to recognize that the evolution of the bubble radius of curvature at the re-entrant neck is NOT affected at all if the sharp neck edge is modified as shown in the lower row of figures in Fig. 6, where the neck now has a finite thickness. During bubble growth, the addition of the vertical section stops the bubble radius of curvature evolution at a certain stage on the lower edge. The bubble then moves to the top edge *with no change in the shape of the interface*. Once the bubble reaches the top edge it completes its evolution exactly as would have happen with the sharp edge cavity neck.

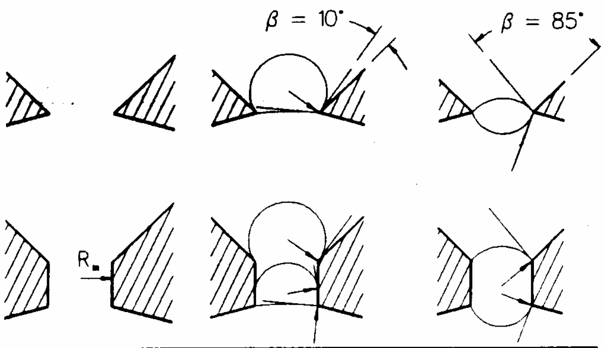


Figure 6. The generalized re-entrant cavity.

### *A. The Generalized Conical - Re-entrant Cavity*

After the previous analysis we can now generalize the shape of the re-entrant cavity by allowing the angles of the walls above and below the neck to have a value from 0° to 360° and from 0° to -360° respectively. The complete picture of the evolution of the bubble radius of curvature is obtained for the case of a re-entrant cavity with a top surface of 180° and a bottom surface of -180° as shown in Fig. 7 (a) and 7 (b), where bubbles of substances of 85° and

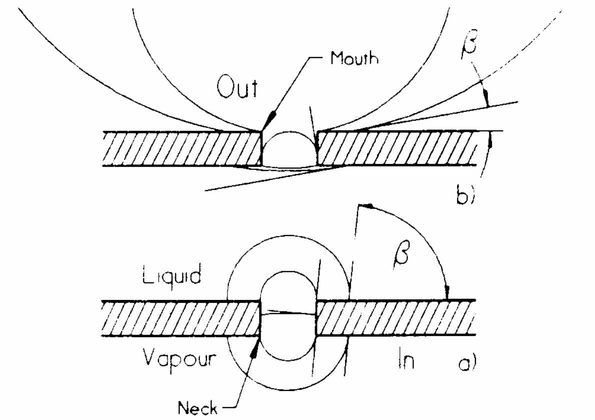


Figure 7. The flat cavity.

10° of contact angle respectively are depicted at several stages of evolution. In this flat cavity the mouth and the neck become equal.

## VI. THE EVOLUTION OF THE BUBBLE RADIUS OF CURVATURE

The evolution of the bubble radius of curvature at the mouth-neck of a generalized re-entrant cavity is shown in Fig. 8 for a substance with a contact angle of 85° (eg. water), and in Fig. 9 for a substance with a contact angle of 10°. In both figures the vertical axis is the log of the absolute value of the bubble radius of curvature. The horizontal axis is the cavity angle $\Theta$ and equals two times the surface orientation $\phi$ ($\Theta = 2\phi$).

A description of the processes shown in the figures is:

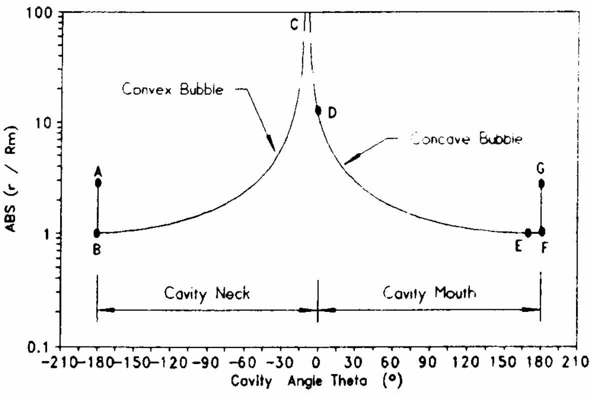


Figure 8. The evolution of r in a generalized re-entrant cavity, for $\beta$ = 85°.

- Processes B -> A occurs when the convex bubble leaves the neck and contracts in volume further *into* the cavity.
- Processes F -> G occurs when the concave bubble leaves the mouth and expands in volume further *out* of the cavity.
- Conditions C exist when the bubble becomes flat and its radius of curvature becomes infinite.
- Condition D corresponds to a cavity angle of 0° (a straight passage normal to the external surface), the turning point between the *inner* and *outer* parts of the re-entrant cavity.
- Condition E is when the bubble radius of curvature equals the cavity mouth size. In both cases shown, this occurs only for concave shaped surfaces.
- The minimum radius of curvature for the convex surfaces occurs at -180° for both substances for this geometric configuration.
- The bubble's volume grows in the A -> G direction

The *absolute minimum* radius of curvature for the *convex* surfaces equals the neck radius, but it could have been reached only if the re-entrant surface would have had values beyond the -180° used in this example.

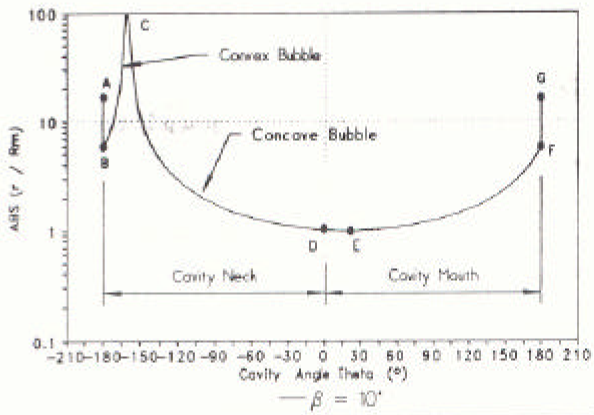


Figure 9. The evolution of r in a generalized re-entrant cavity, for ß = 10°.

## VII. CONCLUSIONS

From figures 8 and 9 its possible to draw the following conclusions:

The requirements for a bubble to remain in thermo-mechanical equilibrium are:

- For a concave bubble, its radius of curvature must shrink when its volume grows, or
- For a convex bubble, its radius of curvature must grow when its volume grows

Then it can be observed that states between B and E are the only equilibrium states for every bubble. States E - G and A - B are unstable, that is, a bubble can not be in equilibrium there, and will either continuously grow or shrink.

The effect of reducing contact angles is to *shift* the graph to the left, towards more negative inner cavity angles.

Since the bubble becomes flat at condition C, it is easily appreciated that if a bubble embryo is to survive at least at saturation conditions, the cavity must have an inner portion (negative values of cavity angle), that is, must be re-entrant.

If the embryo is to survive subcooling, the angle of the re-entrant region must be large.

Poor wetting substances attain saturation at smaller inner cavity angles than well wetting substances.

Because of the shift to the left of the curve for the well wetting substances, it is possible to explain the frequent appearance of TOS at the ONB as a consequence of poor survival capabilities of their bubble embryos, as compared with those of substances with large contact angle.

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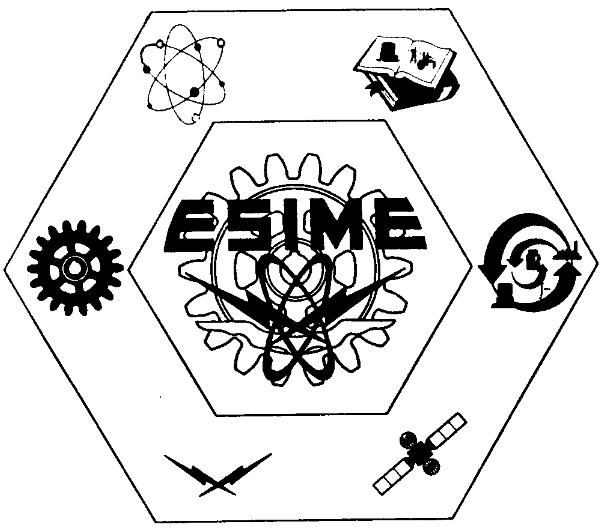


# AVANCES EN INGENIERÍA MECÁNICA

## MEMORIA

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