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Prediction Model of Deflections in PET Fiber Reinforced Concrete Beams

F. J. Baldenebro-Lopez[1,2], J. H. Castorena-Gonzalez[2], J. A. Baldenebro-Lopez[2], J.I. Velazquez-Dimas[3], J. E. Ledezma-Sillas[1], R. Martinez-Sanchez[1] and J. M. Herrera-Ramirez[1]
[1]Centro de Investigación en Materiales Avanzados (CIMAV), Chihuahua, Chih., México
[2]Facultad de Ingeniería Mochis, Universidad Autónoma de Sinaloa, Los Mochis, Sinaloa, México
[3]Facultad de Ingeniería, Universidad Autónoma de Sinaloa, Culiacán, Sinaloa, México

ABSTRACT

The increasing use of polymeric reinforcements in concrete structures requires either the development of a new design theory or the adaptation of current designs considering the engineering properties of this type of materials. In this work a method for calculating the deflections of reinforced concrete elements is proposed, which can be used in predicting the flexural behavior of longitudinally reinforced concrete with PET strips in amounts up to 1%. The model theory assumes that concrete has a tensile load capacity different to zero, characterized by a uniaxial tensile stress-strain diagram. A series of tests were conducted to corroborate the validity of the suggested method, showing that the theory also correctly predicts the creep deformation post-cracking. The deflection results of reinforced concrete with recycled PET strips are presented. The tests are carried out by a simple beam with center-point loading, using three different amounts of reinforcement and comparing the experimental results with the theoretical results of the proposed model.

INTRODUCTION

Concrete is the most widely construction material used in the world due to its high compressive strength, durability and low cost. However, it has inherent disadvantages such as low tensile strength and cracking. Several studies about concrete reinforced with polymeric strips have been made with the aim of improving such deficiencies of the material. The reported results show that it is possible to improve the performance of concrete by reinforcing it with different plastic elements [1]. Reinforced concrete (RC) is a composite material that results from the addition of diverse elements to the brittle matrix of ordinary concrete. Several investigations have been done on the reinforcement of structural materials using plastic elements [2-7]. In the case of concrete, the main reinforcing materials are steel, glass and polymeric fibers. Some of these reinforcements are used in structures such as columns, beams and walls [8-10].

Recently the use of recycled materials as reinforcement in concrete elements has received increasing attention worldwide, being one of them polyethylene terephthalate (PET) obtained from recycled plastic bottles. PET has higher resistance to degradation, lower cost and density, and is also non-conductive of electricity and magnetism. Therefore, polymer reinforced concrete can be an excellent alternative for structures built in or near marine environments or similar corrosive environments (deicing salts), where electromagnetic neutrality and / or electrical insulation are required. Unfortunately, the use of polymers as reinforcements is not free of obstacles that must be solved before application. The main obstacles are the high initial cost, low Young modulus, low tensile strength and the absence of design parameters. The high initial cost is greatly influenced by manufacturing processes; however different methods are currently under development to reduce the cost. The low Young modulus and tensile strength are considered the main engineering disadvantages of PET. This is because PET shows a lower safety margin when compared to its counterpart i.e., reinforced concrete with steel bars. As for the lack of design parameters, more experimental and theoretical data are needed.

It is worth mentioning that the currently available design formulas have been originally developed for steel reinforcement, and nowadays they are adopted by many design codes. However, they are not applicable to the type of reinforcing proposed in this study. Thus, it is necessary to modify the formulas presently used to ensure a better prediction of the deflections in beams manufactured with these materials.

The flexural rigidity of RC beams under service loads is considerably lower than the stiffness calculated on the basis of the uncracked cross-section. This is because the beam contains numerous cracks that become active under tensile stress. However, at the same time, the stiffness is significantly higher than that calculated when the tensile strength of concrete is neglected. This phenomenon, often called tension stiffening is attributed to the fact that the concrete does not break suddenly and completely, but undergoes progressive crack nucleation, which triggers later the effect of crack growth and fracture.

Based on numerous tests [11-17], Branson [18] derived an empirical formula that adequately describes the results of the tests and has been approved by a committee ACI [11]. While this formula is used for practical purposes, it is not derived from the intrinsic properties of the composite material reinforced with fibers, strips or bands. In this article, an approach that includes the characteristics and properties that such reinforcements add to the concrete was developed. The model is able to predict curvatures and deformations beyond the service stress range and the whole development of the ultimate load.

THEORY

Figure 1 shows the arrangement of the reinforcement as well as the cross section of the beam (width b, height H), the assumed deformation profile distribution through the cross section and the corresponding stress distribution. Each row of reinforcement is referenced from the top of the cross section, and its distance is denoted by d_i. For the strain profile, the hypothesis to consider is that there is a linear distribution of the concrete compressive strain (ε_{cc}), concrete tensile strain (ε_{ct}) and strain in a PET layer ($\varepsilon_{pi,\ i=1,2,3}$). The neutral axis (N.A.) position in the beam cross section corresponds to the distance “x” measured from the upper end of the beam cross section (Fig. 1c). In the stress distribution diagram of the beam cross section (Fig. 1d), the force (F_{cc}) acting in the zone subjected to compression stress (above the N.A.) is obtained by integrating the non-linear distribution of the stress acting in this area. In the case of tension (below NA), the acting forces are divided into two. First those provided exclusively by concrete i.e., tension on the higher part (T_{c1}) and tension on the lower part (T_{cc}) and second, those of the reinforcement i.e., tension in each PET layer ($T_{pi,\ i=1,2,3}$). The T_{c1} force is obtained by considering a linear variation of tensile stress as provided by concrete, with a maximum breaking strength value, denoted by the critical stress (f_{cr}) in figure 1c. In the case of the T_{cc} force, a constant stress distribution is assumed. T_c*f_{cr} is the maximum fracture strength.

An important contribution of this research is to propose a value for the coefficient T_c. The magnitude and position of the resulting compressive strength of concrete are obtained by using the stress-strain diagram proposed by Hognestad [19] (Figure 2a). In each of the points to be determined, the procedure starts by assuming the strain value ε_{cc} for the conditions under investigation, and then obtaining the depth of the neutral axis that meets the conditions of forces equilibrium by a trial-and-error approximation. The stress-strain diagram is assumed to fit a parabolic curve as given by Eq. 1 and it is used to calculate the compressive strength of concrete in the plastic behavior region.

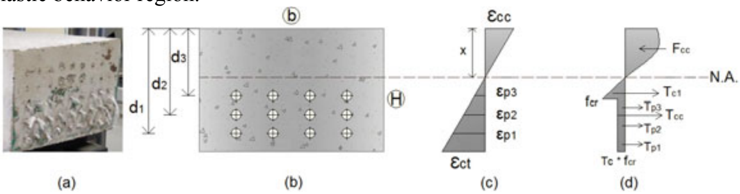
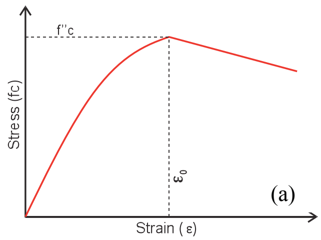


Figure 1. Plot of the internal actuating stresses and strains: (a) cross-section with reinforcement, (b) model schematics of cross-section, (c) strain and (d) stress distribution profile.



$$f_c' = f_c''\left[2\left(\frac{\varepsilon_{cc}}{\varepsilon_0}\right) - \left(\frac{\varepsilon_{cc}}{\varepsilon_0}\right)^2\right] \quad (1)$$

(b)

Figure 2. (a) Stress-strain curve for concrete and (b) corresponding equation as proposed by Hognestad for concrete [20] (ε_0: strain for f'_c of the concrete in the stress-strain curve and f'_c: compressive strength of concrete).

Deformations at each row of reinforcement are calculated by using Eq. 2. In this manner, the tensile stresses corresponding to the experimental stress-strain curve of PET are obtained (Fig. 3). The experimental data are previously obtained by tension tests on the PET strips along the longitudinal direction. An Instron 4469 universal testing machine with a load cell of 50 kN is used with displacement rate of 0.333 mm/s. This procedure results in the internal forces diagram shown in Figure 1 i.e., F_{cc} (Eq. 3), T_{c1} (Eq. 4) where ε_{cr} is the critical deformation, T_{cc} (Eq. 5),where T_{p1}, T_{p2} and T_{p3} (Eq. 6), where A_{ri} is the reinforcement area and f_{ri} is the stress in a PET bands reinforcement layer. Thereby the resisting moment M_r to the deformation ε_{cc} can be calculated with Eq. 7 (see Fig. 4), where Y_{cc} is the distance in between N.A. and F_{cc}, Y_{c1} is the distance in between N.A. and T_{c1}, Y_{cc2} is the distance in between N.A. and T_{cc}.

$$\varepsilon_{pi} = \frac{\varepsilon_{cc}}{x}(d_i - x) \quad (2)$$

$$T_{cc} = T_c f_{cr} b\left[H - \left(1 + \frac{\varepsilon_{cr}}{\varepsilon_{cc}}\right)x\right] \quad (5)$$

$$F_{cc} = f_c' * b * \left[\frac{x\varepsilon_{cc}}{\varepsilon_0}\right]\left[1 - \frac{\varepsilon_{cc}}{3\varepsilon_0}\right] \quad (3)$$

$$T_{pi} = A_{ri} f_{ri} \quad , i = 1,2,3,.... \quad (6)$$

$$T_{c1} = \frac{1}{2}\frac{f_{cr} x \varepsilon_{cr} B}{\varepsilon_{cc}} \quad (4)$$

$$M_r = F_{cc}Y_{cc} + T_{c1}Y_{c1} + T_{cc}Y_{cc2} + \sum_i^3 T_{pi}(d_i - x) \quad (7)$$

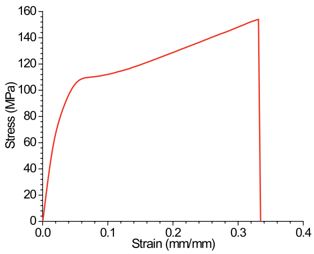


Figure 3. Experimental stress-strain curve of PET.

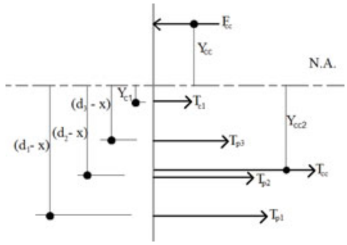


Figure 4. Distribution of forces

The generation of points of the load-deflection plot depends on the load conditions and support of the beam to be studied. In this work a simply supported beam with point center-point loading is considered (Fig. 5). Thus the parameters to be followed are the critical load P_r (Eq. 8) where L is the length between supports of the beam; a calculated curvature factor *curv* (Eq. 9), rigidity of the section *E*Ir* (concrete Poisson modulus E* moment of Inertia *Ir*) (Eq. 10) and the deflection of the beam at the center of the load δ_r (eq. 11). Figure 6 shows the algorithm of the calculation method of Pr and δ used in this investigation.

$$P_r = \frac{4M_r}{L} \quad (8)$$

$$EI_r = \frac{M_r}{curv} \quad (10)$$

$$curv = \frac{\varepsilon_{cc}}{x} \quad (9)$$

$$\delta_r = \frac{P_r L^3}{48 EI_r} \quad (11)$$

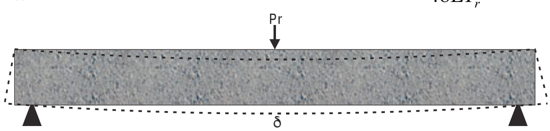


Figure 5. Configuration of the supports, load and deflection in the beam studied.

DISCUSSION

According to Fig. 1, the aim of this work is to determine the value of the coefficient T_c which, when multiplied by the rupture stress (f_{cr}), gives the tensile stress distribution in the concrete. This distribution is considered in the equilibrium equations of resisting moments and forces in concrete beams reinforced with recycled PET strips (Eqs. 2-11). Figure 7 shows experimental and numerical results corresponding to the reinforced beam with bands of recycled PET at 0.25% in volume ratio (B-0.25). It is observed that the value of the theoretical and experimental load to first crack are very similar; the value of the theoretical deflection at this load is 0.80 mm higher in comparison to the experimental, however the difference in the value of the maximum deflection increases in 3 mm. The behavior of the beams containing 0.50% of PET (B-0.50) indicates that the load and deflection to the first crack are very similar, while the calculated maximum deflection is less than the experimental for 0.2 kN (Fig. 8). For the case of the bending behavior of the beams with 1.00% of PET (B-1.00), Fig. 9, the load at the first experimental crack is higher in 0.75kN compared with the theoretically calculated. In the other hand, in the maximum deflection the experimental is greater than that calculated by 9 mm. Experimental and numerical results corresponding to the beams with different ratios (ρ) of recycled PET bands reinforcement, 0.0025 (0.25 vol %), 0.0050 (0.50 vol %) and 0.0100 (1.00 vol %), show that generally for all the beams a very good way of elastic and plastic behavior is predicted. Table 1 presents the maxim loads obtained in each case, finding that at greater value of ρ, the percentage error in the approximation is increased, although the model continues predicting a very good way of the flexural behavior of beams reinforced with PET bands subject to flexion, for three types of beams to Tc factor is 0.41.

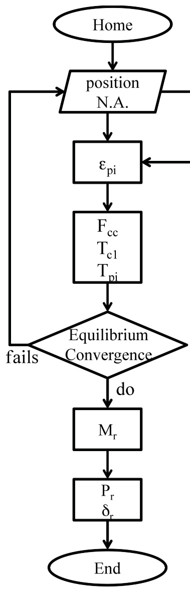


Figure 6. Algorithm of the proposed method.

Table 1. Theoretical and experimental loads.

Beam	ρ	Results	Load max (kN)	% Error
B-0.25	0.0025	Theoretical	7.78	1.64
		Experimental	7.91	
B-0.50	0.0050	Theoretical	9.48	4.06
		Experimental	9.11	
B-1.00	0.0100	Theoretical	14.55	5.43
		Experimental	13.80	

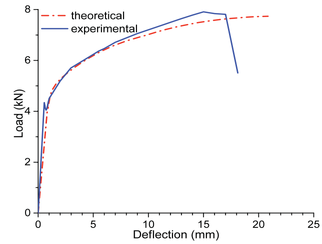


Figure 7. Numerical and experimental load-deflection plots for a beam B-0.25.

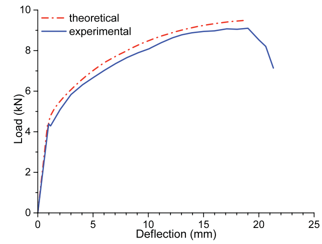


Figure 8. Numerical and experimental load-deflection plots for a beam B-0.50.

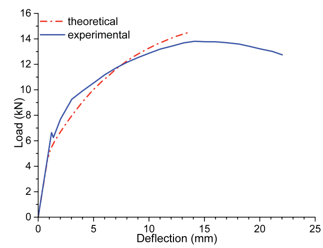


Figure 9. Numerical and experimental load-deflection plots for a beam B-1.00

CONCLUSIONS

It has been shown in the literature and in the present work that the use of recycled PET as reinforcement material, either in the form of short and continuous fibers or strips, substantially improves the strength of concrete beams, as well as their ductility and toughness. However, there is no a model to predict such improvements. In this paper a methodology to predict these enhancements has been found through the following three steps: 1. The application of equilibrium equations, 2. The use of experimental stress-strain curves of PET, and 3. The consideration that the concrete-PET beams have a certain tensile strength at their lower part. A critical stress of 41% of the concrete rupture stress was found to adequately satisfy the theoretical and experimental curves of the specimens tested. The reinforcement percentages used were up to 1% and a condition was to place the PET as continuous strips, in order to have a better control over the percentage of reinforcement and its placement in layers.

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