

Marcuse's power loss model tested for optical fiber coils of small radius.

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Abstract

Recently, the usage of optical fiber coils has increased significantly, especially in the design of physic and chemical sensors. Therefore, it is important to test the theoretical current models developed to predict the power loss throughout optical fiber. In this paper a pioneer and popular model, the Marcuse model of power loss, was studied and evaluated for optical fiber coils of small radii. Power attenuation in a bent fiber data was collected using an Optical Time Domain Reflectometer (OTDR), and it was compared to the theoretical predictions of the Marcuse model. It was observed that the model predicts correctly the attenuation behavior for usual curvature radii, however, it fails to predict accurately the attenuation behavior for small curvature radii, underestimating considerably the actual power loss. Also, it has been observed that at small radii the power loss parameter 2α and the mode propagation constant of the wave guide β stop being constants and become functions of the optical path, particularly of the number of loops in the coil. It is possible that new mechanisms of light leaking are present, due to the extreme distortion of the modes configuration into the fiber at small radii. Those mechanisms cannot be described by a model that considers a power loss parameter 2α , and more specifically the mode propagation constant of the wave guide (β) as constants. Then it is important to develop other models where the previous parameters can be considered as functions of the optical path.

Keywords: Marcuse model; light power attenuation; multimode optical fiber

Introduction

Optical fiber coils have wide applications, especially in the design of physical and chemical sensors. For example, they have been embedded in concrete structures to monitor crack initiation and the effects of different forces and loads when applied to such structures [1]. They can also be installed to detect oil and gas leaks [2-3], and can be used instead of mechanical gyroscopes to stabilize sailing ships [4]. In the aerospace industry [5], they are used to evaluate the rotation speed of an aircraft precisely, etc. Due to the wide application of these sensors, it is important to predict light power loss produced by a coil conformation.

This paper presents an analysis and test of a fundamental model, the Marcuse model [6], which has been the basis for the work of several authors [7-12], to evaluate the power loss behavior in optical fiber coils. Marcuse model calculates an attenuation coefficient (2α) per unit length of a bent optical fiber considering usual curvature radii. In this work, experimental tests were performed and the data obtained was compared to the model predictions for optical fibers rolled up on cylinders, of different radii, a number of times (loops). It is found that the model describes correctly the behavior of attenuation only for usual radii, but fails to do so in curves with smaller curvature radii. Other known model was also examined and it is found to have the same problems as well [11]. Other models have been developed, that are not based on electromagnetic theories, these are phenomenological [13-14].

Marcuse model

The Marcuse model [6] was designed to predict the attenuation of light power in a bent optical fiber. A brief development is presented, for a better understanding of such model. The Marcuse model is based on Maxwell's equations:

$$\nabla \times \nabla \times \mathbf{E} = -\mu \frac{\partial}{\partial t} \nabla \times \mathbf{H} = -\mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (1)$$

$$\nabla(\nabla \cdot \mathbf{E}) - \nabla^2 \mathbf{E} = -\mu \varepsilon \frac{\partial^2 \mathbf{E}}{\partial t^2} \quad (2)$$

Where E and H are the electric and magnetic field strengths; and ε and μ are the electric and magnetic permittivity.

According to Marcuse, the equations of the respective components of the electric field in a cylindrical system are:

$$E_z = A \left[\frac{J_v(ka)}{H_v^{(1)}(i\gamma a)} \right] H_v^{(1)}(i\gamma r') (\cos v\theta) e^{-i\beta z} \quad (3)$$

$$E_\theta = \frac{A\gamma}{2\beta} \frac{J_v(ka)}{H_v^{(1)}(i\gamma a)} \left[H_{v+1}^{(1)}(i\gamma r') \sin(v+1)\theta + H_{v-1}^{(1)}(i\gamma r') \sin(v-1)\theta \right] e^{-i\beta z} \quad (4)$$

Where E_z y E_θ are electric field components; A is the amplitude of the wave; J_v , is the Bessel function; $H_v(1)$ is the Hankel function of the first kind and order v; a is the fiber radius, k is the propagation constant of a plane wave, γ is the cladding field decay rate, β is the mode propagation constant.

Then, the power loss parameter per unit length, 2α can be calculated by:

$$2\alpha = \frac{\sqrt{\pi} \kappa^2 \exp\left[-\frac{2}{3}(\gamma^3 / \beta^2)R\right]}{e_v \gamma^{3/2} V^2 \sqrt{R} K_{v-1}(\gamma a) K_{v+1}(\gamma a)}, \quad e_v = \begin{cases} 2, v = 0 \\ 1, v \neq 0 \end{cases} \quad (5)$$

Where κ is the the core field decay rate R is the curvature radius and V is a normalized frequency parameter. Eq. (6) is the Marcuse model.

$$2\alpha = \frac{2ak^2 e^{2\gamma a} \exp\left[-\frac{2}{3}\left(\frac{\gamma^3}{\beta_g^2}\right)R\right]}{e_v \sqrt{\pi}RV^2} \quad (6)$$

Experimental set up

The equipment used to characterize the power losses was an Optical Time Domain Reflectometer (OTDR), model FM8513, from Tektronics. The fibers were rolled up in the form of coils (varying from 1 through 10 loops) around steel cylinders of 18 different diameters (in the range of 1500 to 10000 micrometers, increasing in steps of 500 micrometers). In those tests, two fiber reels were employed, as shown on Figure 1. The tip of the fiber rolled up on one of them was connected to the OTDR and the other remained free. The power loss was evaluated through the refractogram produced only by the coil, then the losses produced by reels and connectors were discarded the experimental data. A multimode optical fiber, from Thorlabs was used. The fiber and laser parameters characteristics are listed in Table 1.



Figure1. Experimental set up. a) Optical fibers rolled up in a steel cylinder.

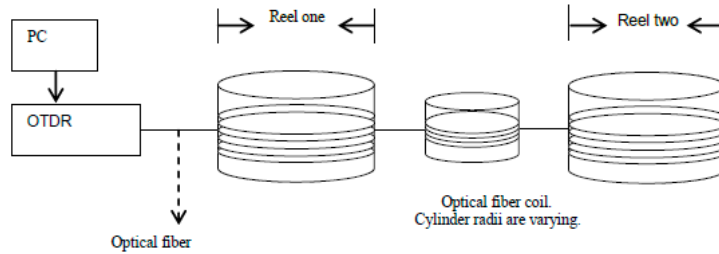


Figure1. Experimental set up. b) Diagram of the experimental set up.

Parameters	Optical Fiber	Laser
Core Diameter	$62.0 \pm 0.7 \mu\text{m}$	
Cladding Diameter	$245.0 \pm 5 \mu\text{m}$	
Numerical Aperture	0.275 ± 0.015	
Wavelength		850 nm.
Pulse Width		10 μs /1000 m.
Output		26.5 dB

Table 1. Parameters: Optical Fiber and Laser

Experimental results

In Figure 2, it can be observed that the smaller the cylinder curvature radius is, the larger the power losses are. Actually, the power loss increases significantly faster when the radius of curvature is smaller than $6000 \mu\text{m}$. However, the power loss does not increase nor decrease linearly with radius. Indeed, the power loss does not double with two loops, neither triples with three loops. The losses increase only a rough 33% with each loop, and it rests almost constant after 6 loops. In the figure 3 appears the error bar graphic.

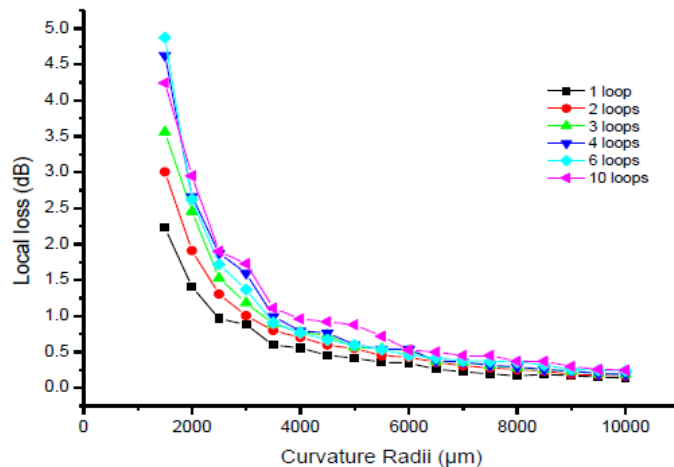


Figure 2. The attenuation behavior of power loss in a coiled optical fiber with several loops.

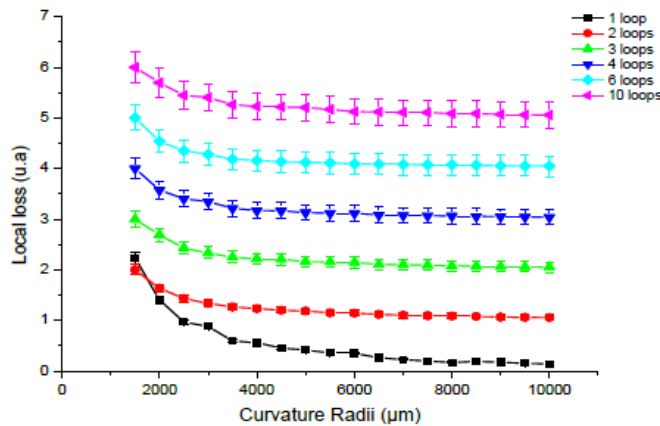


Figure 3. Error bar graphic of power loss in a coiled optical fiber with several loops.

Furthermore, in order to obtain the values of the power loss parameter 2α the values of the previous losses were divided by the fiber length coiled in each cylinder. Then, we compare the experimental data with the predicted values of the Marcuse model. The results are shown in Figure 4. First of all, it is noted that; unexpectedly, for shorter radii to 6000 μm the power loss parameter is not a constant but a function of the number of loops (or more properly a function of the optical path.) Secondly, the

parameter 2α calculated for a single loop is larger than those calculated for additional loops. In the figure 5 appears the error bar graphic.

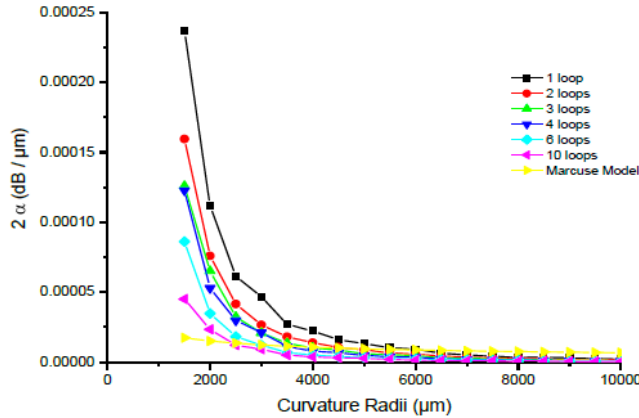


Figure 4. Comparison between the Marcuse model predictions of power loss and the experimental data in a coiled optical fiber with several loops. 2α is the power loss coefficient.

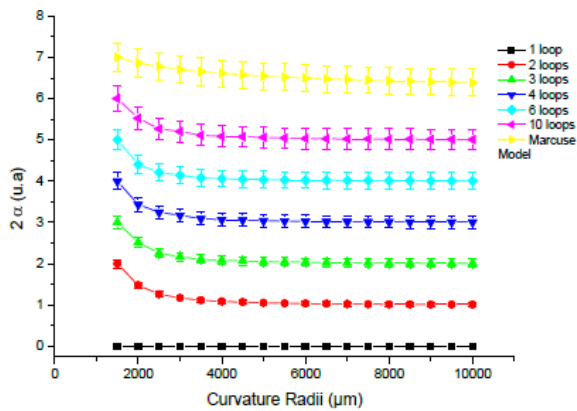


Figure 5. Error bar graphic of the results of the Comparison between the Marcuse model predictions of power loss and the experimental data in a coiled optical fiber with several loops.

These last phenomena are better illustrated in Figure 6, where the 2α values, from the tests performed with cylinders of $1500\ \mu\text{m}$, are plotted as a function of the number of coil loops (this specific radius as chosen because it is the smaller tested, and where the attenuation varies significantly when the number of loops is increased). The theoretical value of 2α from the Marcuse model is also plotted for comparison. It is observed that the largest 2α value corresponds to the first loop, and then this value decreases

constantly. In contrast, the 2α from the Marcuse model remains constant, as expected. It is also interesting to observe that the experimental value and Marcuse model data coincide when the radii is larger than $6000 \mu\text{m}$.

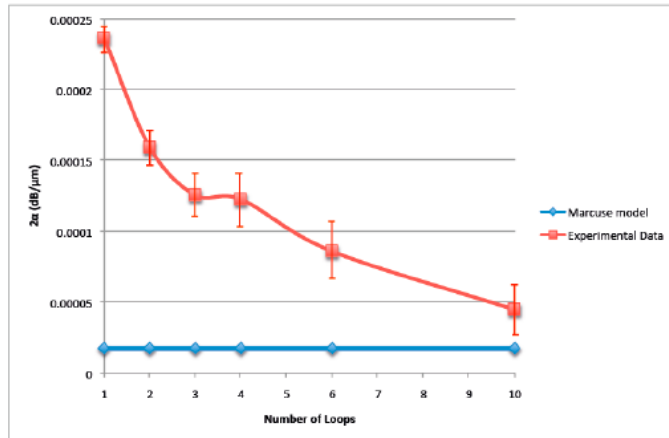


Figure 6. Diagram showing the attenuation parameter for coils with a radius of $1500 \mu\text{m}$.

The model presented by Cole and Schermen [11] was as well revised for the purposes of this document. This model implements several corrections. They corrected some simplifying assumptions made by Marcuse in order to improve the accuracy of the model. Here, the propagation constant β is assumed to change with position. This new model improved the accuracy of the Marcuse model for non-critical radii of curvature. However, this new model does not predict the significant increment in power loss in small radii of curvature as it can be seen in Figure 7.

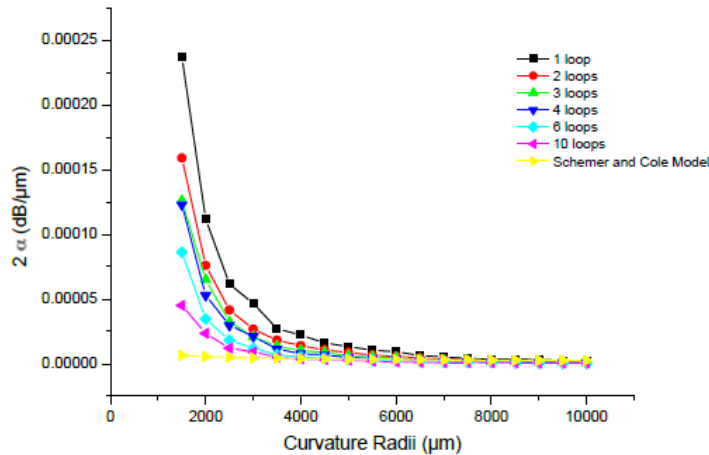


Figure 7. Comparison between the Schermer and Cole model predictions of power loss and the experimental data in a coiled optical fiber with several loops. 2α is the power loss coefficient.

Discussion

Until now, in our knowledge, no author has published any study about the power loss in optical fiber coils. Indeed, the attenuation due to the bending of an optical fiber has been restricted to a maximum of one loop, but never to a number of loops.

The results presented here seem to indicate that there are new mechanisms affecting light leakage in a coiled fiber with a number of loops. Those mechanisms produce, at the first loop, a power loss parameter value of 2α much larger than theoretically predicted, when the radii are smaller than a critical value (in the present case smaller than $6000 \mu\text{m}$). However, the rest of the light power still confined in the fiber seems to leak more slowly on subsequent loops, producing the decrease of the previous parameter.

It is important to mention that; as described by Schermer and Cole [11], the bending tends to distort the fiber modes, and causes them to shift away from the center of curvature. Also, the orientations of the LP modes shifted upon bending, and rotational symmetry was destroyed. Finally, and more importantly, some modes start to interact

and merge with each other. Such profound changes in the fiber mode distributions generate significant changes in the modal propagation constants compared to the straight or quasi-straight fibers. Due to this behavior we cannot continue considering the power loss parameter 2α as a constant for small radii, but a more complex function is necessary to describe this parameter.

Concerning the Marcuse model, in the literature there are a number of papers that improve the precision of this model, maybe the most important one being the work of Schemer and Cole [11], but anyone is capable to describe the behavior of the previous experimental data. Moreover, it is important to remark in Eq. 6 that the modes configuration is taken into account in the mode propagation constant of the wave guide (β). However, as a new modes configuration is probably produced at small radii, this parameter stops being a constant. Nevertheless, once this new arrangement is settled, the rate of light leaking for further loops is fixed and a new value of 2α can be determined. Indeed, in Figure 4 the value of 2α seems to tend to a new constant level. In a next contribution, we will introduce a model where this phenomenon is considered.

Conclusions

In this work the Marcuse model was tested as a tool to predict the attenuation behavior in coiled optical fiber with different radii of curvature. It was found that this model only predicts accurately the power loss for non-critical radii. Indeed, for smaller radii values to $6000 \mu\text{m}$, the model underestimates the 2α values. In fact, it is possible that new mechanisms of light leaking are present, due to the extreme distortion of the modes configuration into the fiber at small radii. Those mechanisms cannot be described by a model that considers the power loss parameter 2α , and more specifically

the mode propagation constant of the wave guide (β) as constants. Then it is important to develop other models where the previous parameters can be considered as functions of the optical path.

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